

$$1. (A \vee (B \leftrightarrow C)) \wedge (A \rightarrow B)$$

$$(T \vee (F \leftrightarrow F)) \wedge (T \rightarrow F)$$

$$(T \vee (T)) \wedge (T \rightarrow F)$$

$$(T \wedge F)$$

$$= \text{False}$$

$$A \vee B \leftrightarrow C \wedge A \rightarrow B$$

$$T \vee F \leftrightarrow F \wedge T \rightarrow F$$

$$T \leftrightarrow F \rightarrow F$$

$$T \leftrightarrow T$$

$$= \text{True}$$

All the brackets are necessary.

- 2 (a) If it snows and whenever it snows, then I eat cake and read newspaper and whenever I read the newspaper then I do not eat cake or it snows, then I read The newspaper

(b)	S	C	N	X	$S \rightarrow X$	$\neg CV S$	$\neg$	γ	Z	A	$A \rightarrow N$
	S	C	N	CAN	$S \rightarrow X$	$\neg CV S$		$N \rightarrow \neg CV S$	$S \wedge (S \rightarrow X)$	$Z \wedge \neg$	
	F	F	F	F	T	T		T	F	F	T
	F	F	T	F	T	T		T	F	F	T
	F	T	F	F	T	F		T	F	F	T
	F	T	T	T	T	F		F	F	F	T
	T	F	F	F	F	T		T	F	F	T
	T	F	T	F	F	T		T	F	F	T
	T	T	F	F	F	T		T	F	F	T
	T	T	T	T	T	T		T	T	T	T

Critical Row

The argument is valid.

3(a) $A \vee B, B \Rightarrow \neg C, C \vdash A$

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|---------------------------|----------|
| 1. $A \vee B$ | |
| 2. $B \Rightarrow \neg C$ | |
| 3. C | |
| 4. $\neg \neg C$ | DN(3) |
| 5. $\neg B$ | M.T(2,4) |
| 6. $B \vee A$ | Com(1) |
| 7. A | D.S(5,6) |

(b) $P \wedge Q, \neg S, \neg R \Rightarrow \neg Q, P \Rightarrow S \vee T \vdash T$

- | | |
|--------------------------------|----------|
| 1. $P \wedge Q$ | |
| 2. $\neg S$ | |
| 3. $\neg R \Rightarrow \neg Q$ | |
| 4. $P \Rightarrow S \vee T$ | |
| 5. $R \vee \neg Q$ | Impl(3) |
| 6. $\neg P \vee (S \vee T)$ | Impl(4) |
| 7. $(\neg P \vee S) \vee T$ | Assoc(6) |
| 8. P | Eimp(1) |
| 9. $S \vee T$ | M.P(4,8) |
| 10. T | D.S(2,9) |

4 (a) $W = I \text{ water my plants}$
 $G = \text{My plants grow}$
 $B = \text{My plants bloom}$
 $H = \text{I am happy.}$

$W \rightarrow (G \wedge B)$
 $G \rightarrow H$
 $\neg H \rightarrow \neg B$

(b) $(W \rightarrow (G \wedge B)), (G \rightarrow H), (\neg H \rightarrow \neg B) \vdash (\neg H \rightarrow \neg P)$ if and only if $(W \rightarrow (G \wedge B)), (G \rightarrow H), (\neg H \rightarrow \neg B), \neg H \vdash \neg P$

$(W \rightarrow (G \wedge B)), (G \rightarrow H), (\neg H \rightarrow \neg B), \neg H \vdash \neg P$

$W \rightarrow (G \wedge B)$	Hypothesis
$G \rightarrow H$	Hypothesis
$\neg H \rightarrow \neg B$	Hypothesis
$\neg H$	Hypothesis

(c) $2^4 = 16 \text{ rows}$

5(a) $M \cap N = M$ if and only if $M \subseteq N$

M	N	$M \cap N$	$M \subseteq N$	$M \cap N = M$
0	0	0	1	1
0	1	0	1	1
1	0	0	0	0
1	1	1	1	1

(b)

M	N	$M \cup N$	$M \cup N = N$	$M \cap N = M$
0	0	0	1	1
0	1	1	1	1
1	0	1	0	0
1	1	1	1	1

$$6. \sum_{k=0}^n 2^k = 2^{n+1} - 1$$

$$\text{For } n=0, \quad 2^0 = 2^{0+1} - 1 = 1$$

$$S(n+1) = \sum_{k=0}^{n+1} 2^k = 2^{n+2} - 1$$

$$\sum_{k=0}^{n+1} 2^k = \sum_{k=0}^n 2^k + 2^{n+1}$$

$$= (2^{n+1} - 1) + 2^{n+1}$$

$$= 2 \cdot 2^{n+1} - 1$$

$$= 2^{n+2} - 1$$

$$= 2^{(n+1)+1} - 1 = 2^{n+1} - 1$$

$S(n+1)$ is true

$$7. F_0 + F_1 + F_2 + \dots + F_n = F_{n+2} - 1$$

$$F_0 + F_1 + F_2 + \dots + F_n + F_{n+1} = (F_0 + F_1 + F_2 + \dots + F_n) + F_{n+1}$$

$$F_0 + F_1 + F_2 + \dots + F_n + F_{n+1} = (F_{n+2} - 1) + F_{n+1}$$

$$F_0 + F_1 + F_2 + \dots + F_n + F_{n+1} = (F_{n+1} + F_{n+2}) - 1$$

$$F_0 + F_1 + F_2 + \dots + F_n + F_{n+1} = F_{n+3} - 1$$

$$= F_{(n+1)+2} - 1$$